1. $(1 + 3i)\,\bar{z} + 2 = 2z - 4i \iff (1 + 3i)(x - iy) + 2 = 2x + 2iy - 4i$ avec $z = x + i\,y$ (forme alg.)

$\iff (x + 3y + 2) + (3x - y)\,i = 2x + i\,(2y - 4)$

$\iff \begin{cases} x + 3y + 2 = 2x \\ 3x - y = 2y - 4 \end{cases}$

$\iff \begin{cases} -x + 3y = -2 \\ 3x - 3y = -4 \end{cases}$

$\iff$ … (résolution du système par substitution ou combinaisons)

$\iff \begin{cases} y = -\dfrac{5}{3} \\ x = -3 \end{cases}$

d'où $S = \{ -3 - \frac{5}{3}\,i \}$

2. $\forall\, z \neq 5 \quad \dfrac{z - 3i}{5 - z} = 1 - i \iff z - 3i = (5 - z)(1 - i)$

$\iff z - 3i = 5 - 5i - z + iz$

$\iff (2 - i)z = 5 - 2i$

$\iff z = \dfrac{5 - 2i}{2 - i} = \dfrac{(5 - 2i)(2 + i)}{(2 - i)(2 + i)} = \dfrac{12 + i}{5} = \dfrac{12}{5} + \dfrac{1}{5}\,i$

d'où $S = \{ \frac{12}{5} + \frac{1}{5}\,i \}$

3. Posons $Z = z^2$

$(E3) \iff z^4 + z^2 - 20 = 0 \iff Z^2 + Z - 20 = 0 \iff Z = 4$ ou $Z = -5$ $(\Delta = 81)$

On cherche donc z tel que $z^2 = 4$ ou $z^2 = -5$

Donc $S = \{ 2\,;\, -2\,;\, i\sqrt{5};\, -i\sqrt{5} \}$

4. $\forall\, z \neq 2 \quad \dfrac{i\,z^2 - 4}{z - 2} = 2 \iff i\,z^2 - 4 = 2z - 4$

$\iff i\,z^2 - 2\,z = 0$

$\iff z\,(i\,z - 2) = 0$ → produit nul

$\iff z = 0$ <u>ou</u> $i\,z - 2 = 0$ donc $z = \dfrac{2}{i} = -2\,i$

$S = \{ 0\,;\; -2\,i \}$

5. $z\,\bar{z} - z^2 = 8 \iff (x^2 + y^2) - (x + iy)^2 = 8$ en posant $z = x + i\,y$ (forme algébrique)

$\iff (x^2 + y^2) - (x^2 - y^2 + 2xy\,i) = 8$

$\iff 2y^2 - 2xy\,i = 8$

$\iff \begin{cases} y^2 = 4 \\ 2xy = 0 \end{cases}$

$\iff \begin{cases} y = 2 \text{ ou } y = -2 \\ x = 0 \text{ ou } y = 0 \end{cases}$

Or $y = 0$ impossible car $y = 2$ ou $y = -2$

d'où $x = 0$ et $y = 2$ ou -2 ainsi $S = \{ 2i\,;\, -2i \}$